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Effect of Intrinsic Decoherence on Nonclassical Effects of the Nondegenerate Bimodal Multiquanta JCM

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We study the nondegenerate multiquanta Jaynes-Cummings model governed by Milburn equation. This models the decoherence of a quantum system as it evolves through intrinsic mechanisms beyond conventional quantum mechanics governed by the Schrödinger equation. We find an exact solution of this equation and apply it to investigate the effects of the intrinsic decoherence on nonclassical effects of the system, such as collapses and revivals of the population inversion and squeezing of the radiation field, for the resonant and the off-resonant cases when the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition state.

KEY WORDS: intrinsic decoherence; nondegenerate; bimodal; multiquanta; JCM.

1. INTRODUCTION

Because of the recent advances in cooling and trapping of ions (Diedrich *et al.*, 1989; Monroe *et al.*, 1995) the motion of the center-of-mass (c.m.) of trapped ions has to be dealt with quantum mechanically. Laser irradiation (Blockley *et al.*, 1992; Blockley and Walls, 1993; Cirac *et al.*, 1993a,c; de Matos Filho and Vogel, 1994, 1996a,b; Vogel and de Matos Filho, 1995) is used to control this motion coherently by coupling the ion's external and internal degrees of freedom. Models have been constructed to describe a two-level ion undergoing quantized vibrational motion within a harmonic trapping potential and interacting with a classical light field (Blockley *et al.*, 1992; Blockley and Walls, 1993; Cirac *et al.*, 1992, 1993a,b,c). It has been pointed out that the dynamics of a trapped ion can be described by a Hamiltonian similar to a Jaynes-Cummings model (JCM) (Jaynes and Cummings, 1963) or its generalizations under certain regimes (Buzek *et al.*, 1997; de Matos Filho and Vogel, 1994, 1996a; Gou *et al.*, 1996; Steinbach *et al.*,

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1997; Vogel and de Matos Filho, 1995). Within the framework of these JCM-like models, various aspects of the dynamics of trapped ions have been studied. For example, quantum nondemolition measurement of vibrational quanta of trapped ions has been analyzed theoretically (de Matos Filho and Vogel, 1996c) and several schemes have been proposed (Bardroff *et al.*, 1996; D'Helon and Milburn, 1996; Poyatos *et al.*, 1996; Wallentowitz and Vogel, 1995) for the reconstruction of quantum–mechanical vibrational states of a trapped ion.

One of these schemes has been successfully applied to the experimental reconstruction of the Wigner function of nonclassical states of the vibrational mode of a trapped ion (Leibfried *et al.*, 1996). It is to be noted that ion trap experiments suffer from decoherence because of classical noise in the laser beams and trapping potential. Such effect has been seen in recent experiments (Itano *et al.*, 1997; Meekhof *et al.*, 1996). This kind of decoherence may be described using the intrinsic decoherence models (Chen and Kuang, 1994; Kuang and Chen, 1994, 1995; Milburn, 1991; Moya-Cessa *et al.*, 1993; Obada *et al.*, 1998, 1999).

The intrinsic decoherence approach has been proposed and investigated in the framework of several models (Caves and Milburn, 1987; Diosi, 1989; Ellis *et al.*, 1989, 1990; Ghirardi *et al.*, 1986, 1990a,b). In particular, Milburn (Milburn, 1991, 1993) proposed a simple intrinsic decoherence models based on an assumption that on sufficiently short time steps the system evolves in a stochastic sequence of identical unitary transformations. This assumption modifies the von Neumann equation for the density operator of a quantum system through a simple modification of the usual Schrödinger evolution equation. The off-diagonal elements of the density operator in Milburn's model are intrinsically suppressed in the energy eigenstate basis, thereby intrinsic decoherence is realized without the usual dissipation associated with the normal decay. The decay is entirely of phase dependence only. Free evolution of a given quantum system has been discussed early (Milburn, 1991) but investigations of interacting subsystems followed (Chen and Kuang, 1994; Kuang and Chen, 1994; Kuang *et al.*, 1995; Moya-Cessa *et al.*, 1993). The latter were concerned with the Jaynes-Cummings model either with one-photon or multiphoton transitions. The Jaynes-Cummings model (JCM) (Jaynes and Cummings, 1963) in quantum optics describes many pure quantum phenomena, such as collapses and revivals of the atomic inversion and oscillations of photon number distribution. It has been generally accepted that these nonclassical effects originate to form quantum coherences between the amplitudes. Therefore, it is an interesting topic to investigate the effects of the intrinsic decoherence on the nonclassical properties in the JCM, when we have two modes of the interacting field affecting the interaction, and hence nondegenerate bimodal multiquanta JCM (Abdalla *et al.*, 1990, 1991; Buzek *et al.*, 1997; Gerry and Eberly, 1990; Gou, 1989; Steinbach *et al.*, 1997). Such a model is discussed in this paper when it is governed by the Milburn equation. On the other hand, there has been increased interest in the problem of decoherence in quantum mechanics because of its possible applications in quantum measurement processes and quantum computers (Chuang and Yamamoto, 1997; Shor, 1995).

Decoherence due to normal decay is often said to be the most efficient effect in physics, to a point where observation comes too late after the effect has reached completion (Omne's, 1997). The effect in action has been observed in quantum optics where the decoherence phenomena transforming a Schrödinger-cat into a statistical mixture was observed while unfolding (Brune *et al.*, 1996). It is well known that JCM in quantum optics (Shore and Knight, 1993) and cavity QED with cold trapped ions (Buzek *et al.*, 1997) can describe many pure quantum phenomena, called nonclassical effects, such as collapses and revivals of population inversion, oscillations of number distributions for quanta, and squeezing of the cavity field.

In this paper, we study this problem for the nondegenerate bimodal multiquanta JCM (Abdalla *et al.*, 1990, 1991; Buzek *et al.*, 1997; Gerry and Eberly, 1990; Gou, 1989; Steinbach *et al.*, 1997) governed by the Milburn model. It will be shown that the intrinsic decoherence in the particle (atom or trapped ion) field interaction modifies the time evolution of the population inversion of the quanta and squeezing of the cavity field. This paper is organized as follows. In section 2, We present the exact solution of the Milburn equation for the nondegenerate bimodal multiquanta Jaynes-Cummings Hamiltonian and give the explicit expression of this solution in the two-dimensional basis of the particle. In section 3, We study the effect of the intrinsic decoherence on population inversion and squeezing of the radiation field in the JCM either in the resonant or the off-resonant cases when the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition state. Finally some concluding remarks are provided.

2. EXACT SOLUTION OF THE MILBURN EQUATION

In standard quantum mechanics the dynamics of a conservative system is described by the density operator $\hat{\rho}(t)$ governed by the evolution operator $\hat{U}(t) = \exp[-\frac{i}{\hbar}t\hat{H}]$, where $\hat{H}$ is the Hamiltonian describing the system. The change in the state of the quantum system in a time interval $(t, t+\tau)$ is given by the unitary transformation

$$\hat{\rho}(t+\tau) = \hat{U}(\tau)\hat{\rho}(t)\hat{U}^{\dagger}(\tau) = \exp\left[-\frac{i}{\hbar}\tau\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar}\tau\hat{H}\right] \quad (1)$$

which is valid for arbitrarily large or small values of τ. Milburn (1991) replaced the above paradigm with three new postulates:

(1) On a sufficiently small time scale the change in the state of the system is stochastic. The probability that the state of the system is changed is $p(\tau)$, which reflects quantum jumps in the state of the system.

(2) Assuming that the state of the system undergoes some changes, then the density operator changes according to the relation

$$\hat{\rho}(t+\tau) = \exp\left[-\frac{i}{\hbar}\hat{\theta}(\tau)\hat{H}\right]\hat{\rho}(t)\,\exp\left[\frac{i}{\hbar}\hat{\theta}(\tau)\hat{H}\right] \tag{2}$$

with $\theta(\tau)$ being some function of τ. In standard quantum mechanics, $p(\tau)$ takes the value 1 while $\theta(\tau) = \tau$. Milburn's proposed it is only required that $p(\tau) \to 1$ and $\theta(\tau) \to \tau$ for values of τ which are sufficiently large.

(3) In Milburn's model it is postulated that

$$\lim_{\tau\to 0} \theta(\tau) = \theta_o \tag{3}$$

which effectively introduces a minimum time step in the universe (Golden, 1992; Kadyshevsky, 1978), whose inverse is equal to the mean frequency of the unitary step, $\gamma = \frac{1}{\theta_o}$. Invoking the assumption that on a very short time scale the probability that the system evolves is $p(\tau) = \gamma\tau$, then the rate of change of $\hat{\rho}(t)$ in Milburn's model satisfies the following equation:

$$\frac{d}{dt}\hat{\rho}(t) = \gamma\left\{\exp\left[-\frac{i}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\,\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right] - \hat{\rho}(t)\right\}. \tag{4}$$

Obviously, the generalized Eq. (4) alters the Schrödinger dynamics. It reduces to the ordinary von Neuman equation for the density operator in the limit $\gamma \to +\infty$. Expanding Eq. (4) to first order in γ^{-1}, the following dynamical equation is obtained:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H}, [\hat{H}, \hat{\rho}]] \tag{5}$$

which is the Milburn equation that we shall study below. This equation has been solved for a harmonic oscillator and a precessing spin system (Milburn, 1991): the simple JCM (Chen and Kuang, 1994; Kuang and Chen, 1994; Moya-Cessa *et al.*, 1993), the resonant multiphoton JCM (Kuang and Chen, 1995), and the nondegenerate two-mode JCM (Obada *et al.*, 1998, 1999). In what follows, we shall consider the exact solution of this equation for the nondegenerate bimodal multiquanta JCM with a detuning parameter when the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition state $|\theta, \phi\rangle$ (Obada and Abdel-Hafez, 1991; Zaheer and Zubairy, 1989).

The nondegenerate bimodal multiquanta JCM consists of a two-level particle (atom or trapped ion) and two modes interacting nonlinearly. The interaction between the particle and the field is affected by k_i quanta of the ith mode. The Hamiltonian for the system, in the rotating wave approximation (Abdalla *et al.*, 1990, 1991; Buzek *et al.*, 1997; Gerry and Eberly, 1990; Gou, 1989; Steinbach *et al.*, 1997), is written as:

$$\hat{H} = \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + \frac{\omega_o}{2}\hat{\sigma}_z + \lambda\left(\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{\sigma}_+\right)$$
$$= \omega_1\left[\hat{n}_1 + \frac{k_1}{2}(\hat{\sigma}_z + I)\right] + \omega_2\left[\hat{n}_2 + \frac{k_2}{2}(\hat{\sigma}_z + I)\right] - \frac{1}{2}(k_1\omega_1 + k_2\omega_2)I$$
$$+ \frac{\Delta}{2}\hat{\sigma}_z + \lambda\left(\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{\sigma}_+\right) \tag{6}$$

where the detuning parameter Δ is given by

$$\Delta = (\omega_o - k_1\omega_1 - k_2\omega_2). \tag{7}$$

This Hamiltonian can be generated from a Raman coupling for an effective 3-level ion in a Λ-configuration confined with a 2-D harmonic trap as described in (Steinbach *et al.*, 1997).

An ion confined in an electromagnetic trap can be regarded as a particle with quantized center-of-mass (c.m.) motion moving in a harmonic potential. Classical laser driving field changes the external states of the ion motion because of exciting or de-exciting internal atomic states of the trapped ion. After using the adiabatic elimination procedure, a general form of this Hamiltonian is obtained. If both the vibrational amplitudes of the ion are much smaller than the laser wavelength, then the Lamb-Dicke limit can be used. In this limit only the leading term in the Lamb-Dicke parameter η whose square gives the ratio of the single photon recoil energy to the energy level spacing in the harmonic oscillator potential. This model [Eq. (6)] can be obtained in the Lamb-Dicke approximation and in the limit of suitable trap anisotropy and specific sideband detunings of the laser. In this case, the $\hat{a}$'s describe vibrational modes and $\hat{\sigma}$'s describe the ion internal states. This Hamiltonian generalizes that of Buzek *et al.* (1997), where one of the $\hat{a}$'s describes the cavity mode and the other describes the vibrational mode of the ion in cavity QED of a trapped ion. As the coupling between the vibrational modes and the external environments is extremely weak, dissipative effects which are inevitable from cavity damping in the optical regime, can be significantly suppressed for the ion motion. For simplicity, in this paper we take $\hbar = 1$ and neglect the constant energy shift $\frac{1}{2}(k_1\omega_1 + k_2\omega_2)I$. Eq. (6) takes the form

$$\hat{H} = \hat{H}_o + \hat{H}_I \tag{8}$$

where

$$\hat{H}_o = \omega_1\left[\hat{n}_1 + \frac{k_1}{2}(\hat{\sigma}_z + I)\right] + \omega_2\left[\hat{n}_2 + \frac{k_2}{2}(\hat{\sigma}_z + I)\right] \tag{9}$$

and

$$\hat{H}_I = \frac{\Delta}{2}\hat{\sigma}_z + \lambda\left(\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{\sigma}_+\right) \tag{10}$$

where $\hat{a}_j(\hat{a}_j^\dagger)$ and $\hat{n}_j = \hat{a}_j^\dagger\hat{a}_j$ are the annihilation (creation) and number operators for the jth mode, λ is the particle-field coupling constant, ω_1 and ω_2 are the field frequencies for the two modes, ω_o is the transition frequency of the particle (atom or trapped ion), $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the "spin flip" operators which satisfy the relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$. Now, we look for the exact solution for the density operator $\hat{\rho}(t)$ of Eq. (5) taking into account the Hamiltonian (6). For convenience, we introduce three auxiliary superoperators (Chen and Kuang, 1994; Kuang and Chen, 1994; Kuang *et al.*, 1995; Moya-Cessa *et al.*, 1993; Obada *et al.*, 1998, 1999) $\hat{J}$, $\hat{S}$, and $\hat{L}$ defined by

$$\exp(\hat{J}\tau)\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\tau}{\gamma}\right)^k \hat{H}^k\hat{\rho}(t)\hat{H}^k \tag{11}$$

$$\exp(\hat{S}\tau)\hat{\rho}(t) = \exp(-i\hat{H}\tau)\hat{\rho}(t)\exp(i\hat{H}\tau) \tag{12}$$

$$\exp(\hat{L}\tau)\hat{\rho}(t) = \exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]\hat{\rho}(t)\exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]. \tag{13}$$

From Eqs. (11–13) it follows that

$$\hat{J}\hat{\rho} = \frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H}, \quad \hat{S}\hat{\rho} = -i[\hat{H}, \hat{\rho}], \quad \hat{L}\hat{\rho} = -\frac{1}{2\gamma}\{\hat{H}^2, \hat{\rho}\} = -\frac{1}{2\gamma}(\hat{H}^2\hat{\rho} + \hat{\rho}\hat{H}^2). \tag{14}$$

By substituting Eq. (14) into Eq. (5), we can obtain the formal solution of the Milburn equation (Srinivas and Daries, 1981; Ueda, 1990; Ueda *et al.*, 1990) as follows:

$$\hat{\rho}(t) = \exp(\hat{J}t)\exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0) \tag{15}$$

where $\hat{\rho}(0)$ is the density operator of the initial particle-field system.

We assume that the two modes of the field are prepared initially in coherent states $|\alpha_1, \alpha_2\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle$ defined by

$$|\alpha_1, \alpha_2\rangle = \sum_{n_1,n_2=0}^{\infty} Q_{n_1}Q_{n_2}|n_1, n_2\rangle = \sum_{n_1,n_2=0}^{\infty} Q_{n_1}Q_{n_2}|n_1\rangle \otimes |n_2\rangle \tag{16}$$

where $Q_{n_i} = e^{-\bar{n}_i/2}\sqrt{\bar{n}_i^{n_i}/n_i!}$, $i = 1, 2$ and the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition state $|\theta, \phi\rangle$ (Obada and

Abdel-Hafez, 1991; Zaheer and Zubairy, 1989), so that

$$\hat{\rho}(0) = \begin{bmatrix} \cos^2 \frac{\theta}{2} |\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| & \cos\frac{\theta}{2}\, \sin\frac{\theta}{2}\, e^{i\phi} |\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| \\ \cos\frac{\theta}{2}\, \sin\frac{\theta}{2}\, e^{-i\phi} |\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| & \sin^2 \frac{\theta}{2} |\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| \end{bmatrix}. \tag{17}$$

In a 2-D basis for the particle the Hamiltonian (6) can be expressed as a sum of $(\hat{H}_o)$, which is diagonal in this basis, and $(\hat{H}_I)$, which is not. It is easy to prove that $(\hat{H}_o)$ and $(\hat{H}_I)$ commute, i.e.

$$[\hat{H}_o, \hat{H}_I] = 0. \tag{18}$$

Thus the representation now takes the form

$$\hat{H}_o = \begin{bmatrix} \omega_1(\hat{n}_1 + k_1) + \omega_2(\hat{n}_2 + k_2) & 0 \\ 0 & \omega_1\hat{n}_1 + \omega_2\hat{n}_2 \end{bmatrix} \tag{19}$$

$$\hat{H}_I = \lambda \begin{bmatrix} \frac{\Delta}{2\lambda} & \hat{a}_1^{k_1}\hat{a}_2^{k_2} \\ \hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & -\frac{\Delta}{2\lambda} \end{bmatrix}. \tag{20}$$

Similarly, the square of the Hamiltonian (6) can also be expressed as a sum of two matrices in the form

$$\hat{H}^2 = \hat{A} + \hat{B}, \quad [\hat{A}, \hat{B}] = 0 \tag{21}$$

where $\hat{A}$ is diagonal in the form

$$\hat{A} = \begin{bmatrix} \hat{\Theta}^2(n_1 + k_1, n_2 + k_2) & 0 \\ 0 & \hat{\Theta}^2(n_1, n_2) \end{bmatrix} \tag{22}$$

and $\hat{B}$ is given by

$$\hat{B} = 2\lambda \begin{bmatrix} \left(\frac{\Delta}{2\lambda}\right) \hat{W}(n_1 + k_1, n_2 + k_2) & \hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{W}(n_1, n_2) \\ \hat{W}(n_1, n_2)\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & -\left(\frac{\Delta}{2\lambda}\right) \hat{W}(n_1, n_2) \end{bmatrix} \tag{23}$$

with

$$\hat{W}(n_1, n_2) = \omega_1\hat{n}_1 + \omega_2\hat{n}_2 \tag{24}$$

$$\hat{\nu}^2(n_1, n_2) = \hat{a}_1^{\dagger k_1}\hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{a}_2^{k_2} = \frac{\hat{n}_1!}{(\hat{n}_1 - k_1)!}\frac{\hat{n}_2!}{(\hat{n}_2 - k_2)!} \tag{25}$$

$$\hat{\mu}^2(n_1, n_2) = \hat{\nu}^2(n_1, n_2) + \left(\frac{\Delta}{2\lambda}\right)^2 \tag{26}$$

and

$$\hat{\Theta}^2(n_1, n_2) = \hat{W}^2(n_1, n_2) + \lambda^2\hat{\mu}^2(n_1, n_2). \tag{27}$$

Taking into account the initial condition (17), we can write down the following expression

$$\begin{aligned}\hat{\rho}_2(t) &= \exp(\hat{S}t)\,\exp(\hat{L}t)\hat{\rho}(0)\\ &= \exp(-i\hat{H}_I t)\,\exp\left(-\frac{t}{2\gamma}\hat{B}\right)\hat{\rho}_1(t)\,\exp\left(-\frac{1}{2\gamma}\hat{B}t\right)\exp(i\hat{H}_I t) \end{aligned}\tag{28}$$

where the auxiliary operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t) = \begin{bmatrix} \cos^2\frac{\theta}{2}|\hat{\Psi}(t)\rangle\langle\hat{\Psi}(t)| & \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,e^{i\phi}|\hat{\Psi}(t)\rangle\langle\hat{\Psi}'(t)| \\ \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,e^{-i\phi}|\hat{\Psi}'(t)\rangle\langle\hat{\Psi}(t)| & \sin^2\frac{\theta}{2}|\hat{\Psi}'(t)\rangle\langle\hat{\Psi}'(t)| \end{bmatrix}\tag{29}$$

with

$$\begin{aligned}|\hat{\Psi}(t)\rangle &= \exp\left[-\frac{t}{2\gamma}\hat{\Theta}^2(n_1+k_1, n_2+k_2)\right]\\ &\quad\times\,\exp[-i\hat{W}(n_1+k_1, n_2+k_2)t]|\alpha_1,\alpha_2\rangle\end{aligned}\tag{30}$$

and

$$|\hat{\Psi}'(t)\rangle = \exp\left[-\frac{t}{2\gamma}\hat{\Theta}^2(n_1, n_2)\right]\exp[-i\hat{W}(n_1, n_2)t]|\alpha_1,\alpha_2\rangle.\tag{31}$$

The powers of the operator $\hat{B}$ can be written as

$$\hat{B}^{2k} = \begin{bmatrix} [2\lambda\hat{W}(n_1+k_1, n_2+k_2)\hat{\mu}(n_1+k_1, n_2+k_2)]^{2k} & 0 \\ 0 & [2\lambda\hat{W}(n_1, n_2)\hat{\mu}(n_1, n_2)]^{2k} \end{bmatrix}\tag{32}$$

$$\hat{B}^{2k+1} = \begin{bmatrix} \frac{\Delta}{2\lambda}\frac{[2\lambda\hat{W}(n_1+k_1,n_2+k_2)\hat{\mu}(n_1+k_1,n_2+k_2)]^{2k+1}}{\hat{\mu}(n_1+k_1,n_2+k_2)} & \hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)} \\ \frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & -\frac{\Delta}{2\lambda}\frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)} \end{bmatrix}\tag{33}$$

then we can write the operator $\exp[-\frac{t}{2\gamma}\hat{B}]$ in the form

$$\begin{aligned}&\exp\left[-\frac{t}{2\gamma}\hat{B}\right]\\ &= \begin{bmatrix} \hat{X}(n_1+k_1, n_2+k_2, t) - \frac{\Delta}{2\lambda}\frac{\hat{Y}(n_1+k_1,n_2+k_2,t)}{\hat{\mu}(n_1+k_1,n_2+k_2)} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ -\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{X}(n_1, n_2, t) + \frac{\Delta}{2\lambda}\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix}\end{aligned}\tag{34}$$

where

$$\hat{X}(n_1, n_2, t) = \cosh\left[\frac{\lambda t}{\gamma}\hat{W}(n_1, n_2)\hat{\mu}(n_1, n_2)\right],$$
$$\hat{Y}(n_1, n_2, t) = \sinh\left[\frac{\lambda t}{\gamma}\hat{W}(n_1, n_2)\hat{\mu}(n_1, n_2)\right]. \tag{35}$$

Similarly, we can write the operator $\exp[-i\hat{H}_I t]$ in the 2-D basis for the particle as

$$\exp[-i\hat{H}_I t]$$
$$= \begin{bmatrix} \hat{C}(n_1+k_1, n_2+k_2, t) - i\frac{\Delta}{2\lambda}\frac{\hat{S}(n_1+k_1,n_2+k_2,t)}{\hat{\mu}(n_1+k_1,n_2+k_2)} & -i\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ i\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{C}(n_1, n_2, t) + i\frac{\Delta}{2\lambda}\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix} \tag{36}$$

with

$$\hat{C}(n_1, n_2, t) = \cos[\lambda t\hat{\mu}(n_1, n_2)] \quad \text{and} \quad \hat{S}(n_1, n_2, t) = \sin[\lambda t\hat{\mu}(n_1, n_2)]. \tag{37}$$

Then,

$$\exp[-i\hat{H}_I t]\exp\left(-\frac{t}{2\gamma}\hat{B}\right)$$
$$= \begin{bmatrix} \hat{R}(n_1+k_1, n_2+k_2, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1+k_1,n_2+k_2,t)}{\hat{\mu}(n_1+k_1,n_2+k)} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ -\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{R}(n_1, n_2, t) + \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix} \tag{38}$$

where

$$\hat{R}(n_1, n_2, t) = \hat{C}(n_1, n_2, t)\hat{X}(n_1, n_2, t) + i\hat{S}(n_1, n_2, t)\hat{Y}(n_1, n_2, t) \tag{39}$$

$$\hat{V}(n_1, n_2, t) = \hat{C}(n_1, n_2, t)\hat{Y}(n_1, n_2, t) + i\hat{S}(n_1, n_2, t)\hat{X}(n_1, n_2, t). \tag{40}$$

Substituting Eqs. (29) and (38) into Eq. (28), we obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\rho}_{11}(t) & \hat{\rho}_{12}(t) \\ \hat{\rho}_{21}(t) & \hat{\rho}_{22}(t) \end{bmatrix} \tag{41}$$

with

$$\hat{\rho}_{11}(t) = \cos^2\frac{\theta}{2}\hat{\Psi}_{11}(t) + \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,e^{-i\phi}\hat{\Psi}_{31}(t)$$
$$+ \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,e^{i\phi}\hat{\Psi}_{13}(t) + \sin^2\frac{\theta}{2}\hat{\Psi}_{33}(t) \tag{42}$$

$$\hat{\rho}_{12}(t) = \cos^2\frac{\theta}{2}\hat{\Psi}_{12}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{-i\phi}\hat{\Psi}_{32}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{i\phi}\hat{\Psi}_{14}(t) + \sin^2\frac{\theta}{2}\hat{\Psi}_{34}(t) \tag{43}$$

$$\hat{\rho}_{21}(t) = \cos^2\frac{\theta}{2}\hat{\Psi}_{21}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{-i\phi}\hat{\Psi}_{41}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{i\phi}\hat{\Psi}_{23}(t) + \sin^2\frac{\theta}{2}\hat{\Psi}_{43}(t) \tag{44}$$

$$\hat{\rho}_{22}(t) = \cos^2\frac{\theta}{2}\hat{\Psi}_{22}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{-i\phi}\hat{\Psi}_{42}(t) + \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ e^{i\phi}\hat{\Psi}_{24}(t) + \sin^2\frac{\theta}{2}\hat{\Psi}_{44}(t) \tag{45}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}(t) = |\hat{\Psi}_i(t)\rangle\langle\hat{\Psi}_j(t)| \quad (i,\ j = 1, 2, 3, 4) \tag{46}$$

with

$$|\hat{\Psi}_1(t)\rangle = \left[\hat{R}(n_1 + k_1, n_2 + k_2, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1 + k_1, n_2 + k_2, t)}{\hat{\mu}(n_1 + k_1, n_2 + k)}\right]|\hat{\Psi}(t)\rangle \tag{47}$$

$$|\hat{\Psi}_2(t)\rangle = \left[-\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\frac{\hat{V}(n_1 + k_1, n_2 + k_2, t)}{\hat{\mu}(n_1 + k_1, n_2 + k)}\right]|\hat{\Psi}(t)\rangle \tag{48}$$

$$|\hat{\Psi}_3(t)\rangle = \left[-\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{V}(n_1, n_2, t)}{\hat{\mu}(n_1, n_2)}\right]|\hat{\Psi}^{'}(t)\rangle \tag{49}$$

and

$$|\hat{\Psi}_4(t)\rangle = \left[\hat{R}(n_1, n_2, t) + \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1, n_2, t)}{\hat{\mu}(n_1, n_2)}\right]|\hat{\Psi}^{'}(t)\rangle \tag{50}$$

where $|\hat{\Psi}(t)\rangle$ and $|\hat{\Psi}^{'}(t)\rangle$ are given by Eqs. (30) and (31). Taking into account the definition of the superoperator $\hat{J}$, we can obtain the action of the operator $\exp(\hat{J}t)$ on the density operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{t}{\gamma}\right)^k \hat{H}^k\hat{\rho}_2(t)\hat{H}^k \tag{51}$$

where the Hamiltonian $\hat{H}$ and the operator $\hat{\rho}_2(t)$ are given by Eqs. (6) and (41), respectively. Equation (51) describes the exact solution of the Milburn equation [Eq. (5)] for the nondegenerate bimodal multiquanta JCM. Once the density operator is calculated all relevant statistical quantities can be computed.

3. EFFECT OF THE INTRINSIC DECOHERENCE ON NONCLASSICAL EFFECTS OF THE SYSTEM

In this section, we investigate the effect of the intrinsic decoherence on nonclassical effects of the nondegenerate bimodal multiquanta JCM for two cases—exact resonant and off-resonant—when the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition state.

3.1. Population Inversion

It is well known that in JCM the quantum coherences built up during the interaction between the field and the particle significantly affect the dynamics of the particle (Abdalla *et al.*, 1990, 1991; Buzek *et al.*, 1992, 1997; Gea-Banacloche, 1991; Gerry and Eberly, 1990; Gou, 1989; Narozhny *et al.*, 1981; Phoenix and Knight, 1988, 1991; Shore and Knight, 1993). The existence of the quantum coherences is the reason why one can observe collapses and revivals of the population inversion of the particle. Now we evaluate the population inversion in the nondegenerate multiquanta JCM. Using the exact solution $\hat{\rho}(t)$, we find that population inversion is given by

$$\begin{aligned}
W(t) = \langle\hat{\sigma}_z(t)\rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{\sigma}_z] = \sum_{n_1,n_2=0}^{\infty}\Bigg[\cos^2\frac{\theta}{2}\frac{|Q_{n_1}|^2|Q_{n_2}|^2}{\mu^2(n_1+k_1,n_2+k_2)}\\
\times\left\{\left(\frac{\Delta}{2\lambda}\right)^2+\nu^2(n_1+k_1,n_2+k_2)\exp\left[-\frac{2\lambda^2t}{\gamma}\mu^2(n_1+k_1,n_2+k_2)\right]\right.\\
\left.\times\cos 2\lambda t\mu(n_1+k_1,n_2+k_2)\right\}+\sin^2\frac{\theta}{2}\frac{|Q_{n_1+k_1}|^2|Q_{n_2+k_2}|^2}{\mu^2(n_1+k_1,n_2+k_2)}\\
\times\left\{-\left(\frac{\Delta}{2\lambda}\right)^2-\nu^2(n_1+k_1,n_2+k_2)\exp\left[-\frac{2\lambda^2t}{\gamma}\mu^2(n_1+k_1,n_2+k_2)\right]\right.\\
\left.\times\cos 2\lambda t\mu(n_1+k_1,n_2+k_2)\right\}+\sin\theta\, Q_{n_1+k_1}Q_{n_2+k_2}Q^*_{n_1}Q^*_{n_2}\\
\times\frac{\nu(n_1+k_1,n_2+k_2)}{\mu(n_1+k_1,n_2+k_2)}\left\{\frac{\left(\frac{\Delta}{2\lambda}\right)}{\mu(n_1+k_1,n_2+k_2)}\cos\phi\left(1-\exp\left[-\frac{2\lambda^2t}{\gamma}\mu^2\right.\right.\right.\\
\left.\left.\times(n_1+k_1,n_2+k_2)\right]\cos 2\lambda t\mu(n_1+k_1,n_2+k_2)\right)-\sin\phi\,\sin 2\lambda t\mu\\
\left.\left.\times(n_1+k_1,n_2+k_2)\,\exp\left[-\frac{2\lambda^2t}{\gamma}\mu^2(n_1+k_1,n_2+k_2)\right]\right\}\Bigg]. \qquad (52)
\end{aligned}$$

We see that when $\theta = 0$ (the particle initially in excited state) the above equations coincide with those of Obada *et al.* (1998), and when $n_2 = 0$, $\Delta = 0$ coincide with those of Moya-Cessa *et al.* (1993).

We discuss the general behaviour of the population inversion for the nondegenerate multiquanta JCM, when the particle (atom or trapped ion) initially starts in a coherent superposition states.

The numerical results are shown in Figs. 1–6 for various values of the parameter $\frac{\lambda^2}{\gamma}$, and different values of the detuning parameter $\frac{\Delta}{2\lambda}$, (namely 0,10), and fixed initial mean numbers of quanta $\bar{n}_1$ and $\bar{n}_2$.

In Figs. 1 and 2 we plotted the population inversion $W(t)$ with ($\theta = 0$) (the particle initially in excited state) for three values of the parameter $\frac{\lambda^2}{\gamma}$ (namely 10^{-6},

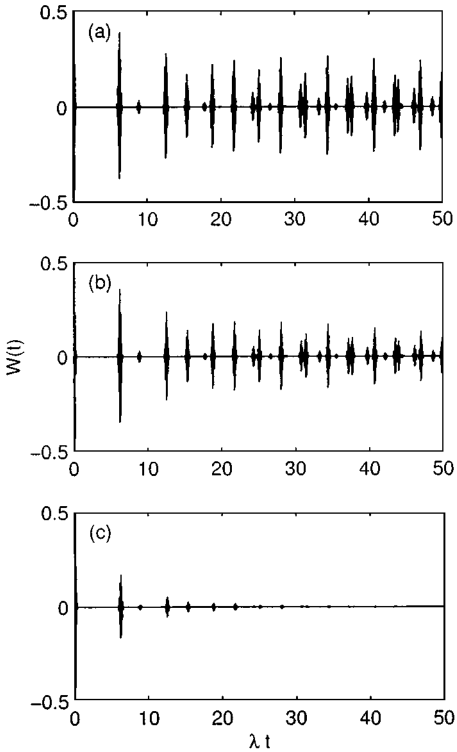

Fig. 1. Time evolution of the population inversion $W(t)$ with $\theta = 0$ (the particle initially in the excited state) interacting with the coherent state $|\alpha_1, \alpha_2\rangle(\bar{n}_1 = \bar{n}_2 = 25)$ for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-5}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-4}$.

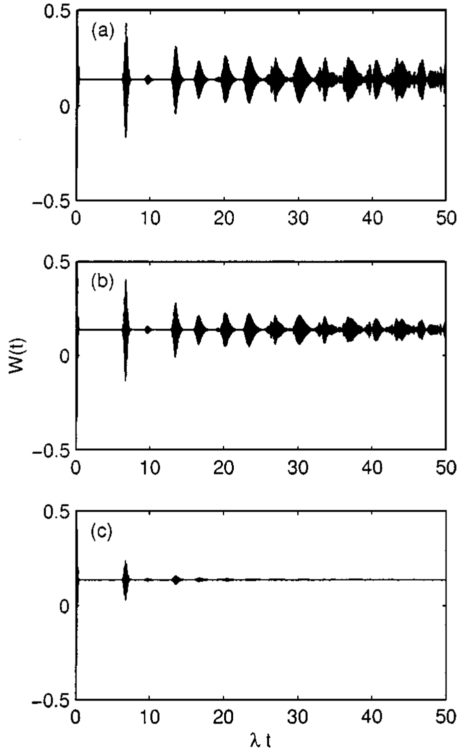

Fig. 2. Same as Fig. 1 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

10^{-5}, 10^{-4}) with the fixed initial mean numbers of quanta $\bar{n}_1 = \bar{n}_2 = 25$ in the two cases—exact resonance i.e., $\frac{\Delta}{2\lambda} = 0$ and the off-resonant case.

Figures 3 and 4 are same as Figs. 1 and 2 but with $\theta = \pi$ (the particle initially in ground state).

Figures 5 and 6 are same as Figs. 1 and 2 but with $\theta = \frac{\pi}{2}$. In these figures (see Figs. 5 and 6), we observe rapid deterioration of revivals of the population inversion than in two cases $\theta = 0$ and $\theta = \pi$.

These figures show that with the decrease of the parameter γ, i.e., with a more rapid suppression of quantum coherences, we can observe rapid deterioration of revivals of the population inversion. Which means that the decay of quantum coherences is due to the very specific time evolution described by Eq. (5), i.e., because of the intrinsic decoherence. The amplitudes of the revivals are suppressed further by increasing the detuning parameter Δ. However as time evolves we observe that $W(t)$ settles to a positive value which means energy is stored in the atomic system.

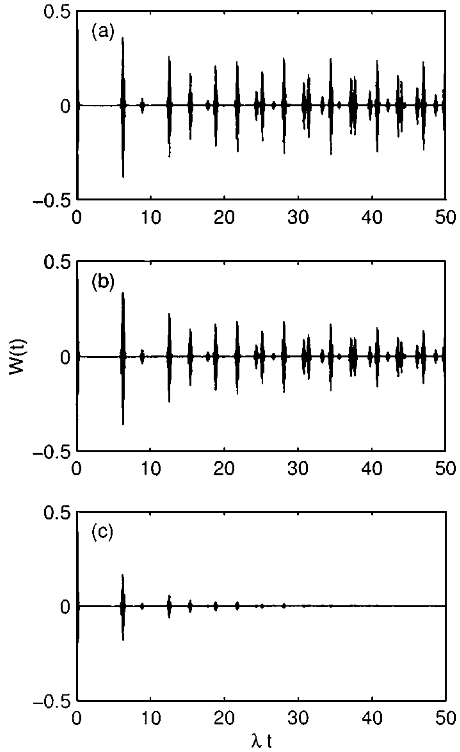

Fig. 3. Same as Fig. 1 but with $\theta = \pi$ (the particle initially in the ground state).

3.2. Amplitude-Squared Squeezing of the Field

Now, we study the amplitude-squared squeezing of the field of the nondegenerate bimodal multiquanta JCM governed by the Milburn equation and discuss effects of the intrinsic decoherence on the squeezing. We define the operators of the real and imaginary parts of the square of the amplitude (Hillery, 1987a,b)

$$\hat{X}_{j1} = \frac{1}{2}\left[\hat{A}_j^2(t) + \hat{A}_j^{\dagger 2}(t)\right], \quad \hat{X}_{j2} = \frac{1}{2i}\left[\hat{A}_j^2(t) - \hat{A}_j^{\dagger 2}(t)\right], \qquad (j = 1, 2) \tag{53}$$

where $\hat{A}_j(t) = \hat{a}_j e^{i\omega_j t}$, $\hat{A}_j^{\dagger}(t) = \hat{a}_j^{\dagger} e^{-i\omega_j t}$, are the slowly varying operators. These operators satisfy the commutation relation,

$$[\hat{X}_{j1}, \hat{X}_{j2}] = i(2\hat{n}_j + 1), \quad (j = 1, 2) \tag{54}$$

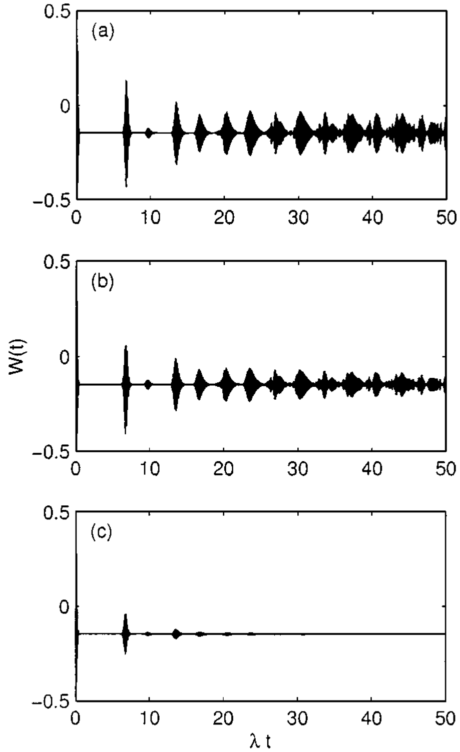

Fig. 4. Same as Fig. 3 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

which implies the uncertainty relation

$$(\Delta\hat{X}_{j1})^2(\Delta\hat{X}_{j2})^2 \geq \frac{1}{4}|\langle[\hat{X}_{j1}, \hat{X}_{j2}]\rangle|^2. \tag{55}$$

The state of the field is said to be amplitude-squared squeezed whenever one of the two quadratures satisfies the relation:

$$(\Delta\hat{X}_{j1\,\mathrm{or}\,2}) < \frac{1}{2}(2\hat{n}_j + 1), \quad (j = 1 \text{ or } 2) \tag{56}$$

where $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$, $(j = 1, 2)$. On the other hand, the condition (56) can be rewritten as

$$S_i^{(j)} = (\Delta\hat{X}_{ji})^2 - \frac{1}{2}(\langle 2\hat{n}_j + 1\rangle), \quad (i = 1, 2,\ j = 1 \text{ or } 2) \tag{57}$$

and squeezing occurs when $S_1^{(j)}$ or $S_2^{(j)} < 0$. In terms of the photon annihilation and creation operators of the field, we get for the amplitude-squared squeezing

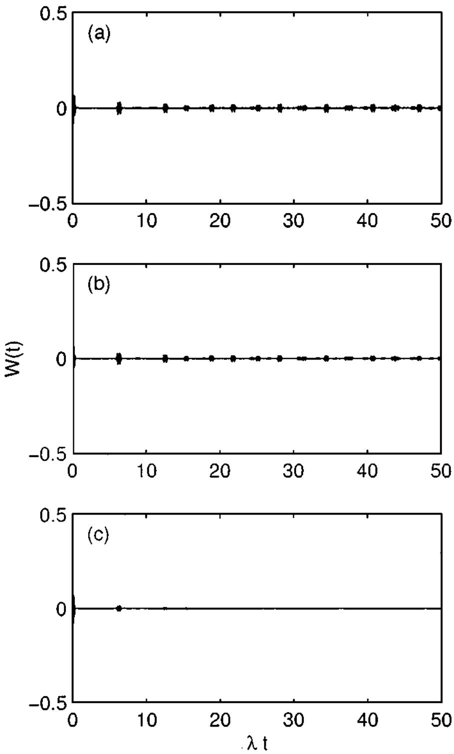

Fig. 5. Time evolution of the population inversion $W(t)$ with $\theta = \frac{\pi}{2}$ interacting with the coherent state $|\alpha_1, \alpha_2\rangle(\bar{n}_1 = \bar{n}_2 = 25)$ for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-5}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-4}$.

factors $S^{(j)}$ the expression

$$S_1^{(j)} = \frac{1}{4}\left[2\langle\hat{A}_j^{\dagger 2}\hat{A}_j^2\rangle + \langle\hat{A}_j^4\rangle + \langle\hat{A}_j^{\dagger 4}\rangle - \left(\langle\hat{A}_j^2\rangle + \langle\hat{A}_j^{\dagger 2}\rangle\right)^2\right] \tag{58}$$

$$S_2^{(j)} = \frac{1}{4}\left[2\langle\hat{A}_j^{\dagger 2}\hat{A}_j^2\rangle - \langle\hat{A}_j^4\rangle - \langle\hat{A}_j^{\dagger 4}\rangle + \left(\langle\hat{A}_j^2\rangle - \langle\hat{A}_j^{\dagger 2}\rangle\right)^2\right]. \tag{59}$$

Now, we can obtain the amplitude-squared squeezing of the first or the second mode from the last expression if we take $j = 1$ or 2. The field density operator $\hat{\rho}_F(t)$ is obtained from $\hat{\rho}(t)$ by tracing over the atomic states,

$$\hat{\rho}_F(t) = \mathrm{Tr}_A \hat{\rho}(t). \tag{60}$$

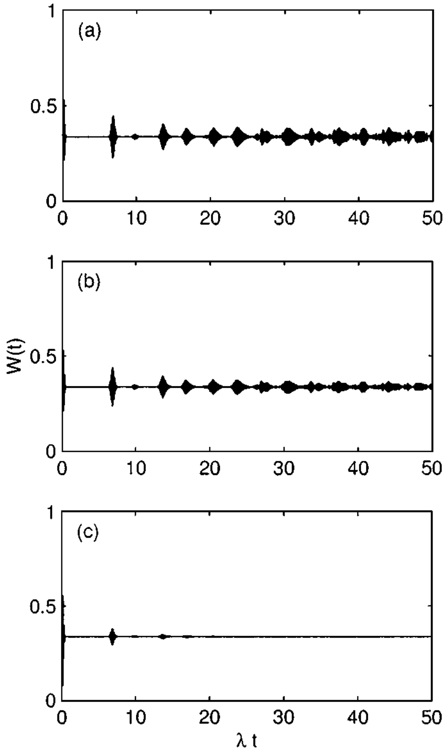

Fig. 6. Same as Fig. 5 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

From the field density operator, the expectation values in the general form for the field operators $\hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}$ is calculated from the formula

$$\left\langle \hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}\right\rangle = \mathrm{Tr}_{\text{field}}\left[\hat{\rho}_F(t)\hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}\right]. \tag{61}$$

The final result is

$$\begin{aligned}
&\left\langle \hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}\right\rangle \\
&\quad = \frac{1}{4}\sum_{n_1,n_2=0}^{\infty}\left[\cos^2\frac{\theta}{2}Q_{n_1+s_1}Q_{n_2+s_2}Q^*_{n_1+r_1}Q^*_{n_2+r_2}\left\{[J(1+D_s)(1+D_r)\right.\right.\\
&\qquad + J_1\cdot L_1]\exp(i\lambda t\mathbf{a}_-(n_1,n_2))\exp\left(-\frac{t}{2\gamma}\mathbf{b}_+(n_1,n_2)\right)\\
&\qquad + [J(1+D_s)(1-D_r) - J_1\cdot L_1]\exp(-i\lambda t\mathbf{a}_+(n_1,n_2))
\end{aligned}$$

$$
\begin{aligned}
&\times \exp\left(-\frac{t}{2\gamma}\mathbf{c}_-(n_1,n_2)\right) + [J(1-D_s)(1+D_r) - J_1\cdot L_1] \\
&\times \exp(i\lambda t\mathbf{a}_+(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{c}_+(n_1,n_2)\right) + [J(1-D_s)(1-D_r) \\
&+ J_1\cdot L_1] \exp(-i\lambda t\mathbf{a}_-(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{b}_-(n_1,n_2)\right)\Bigg\} \\
&+ \sin^2\frac{\theta}{2} Q_{n_1+s_1+k_1} Q_{n_2+s_2+k_2} Q^*_{n_1+r_1+k_1} Q^*_{n_2+r_2+k_2} \Bigg\{[J_1(1-D_s)(1-D_r) \\
&+ J\cdot L_1] \exp(i\lambda t\mathbf{a}_-(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{b}_+(n_1,n_2)\right) \\
&+ [J_1(1-D_s)(1+D_r) - J\cdot L_1] \exp(-i\lambda t\mathbf{a}_+(n_1,n_2)) \\
&\times \exp\left(-\frac{t}{2\gamma}\mathbf{c}_-(n_1,n_2)\right) + [J_1(1+D_s)(1-D_r) - J\cdot L_1] \\
&\times \exp(i\lambda t\mathbf{a}_+(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{c}_+(n_1,n_2)\right) + [J_1(1+D_s)(1+D_r) \\
&+ J\cdot L_1] \exp(-i\lambda t\mathbf{a}_-(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{b}_-(n_1,n_2)\right)\Bigg\} \\
&+ \cos\frac{\theta}{2}\sin\frac{\theta}{2} e^{-i\phi} Q_{n_1+s_1+k_1} Q_{n_2+s_2+k_2} Q^*_{n_1+r_1} Q^*_{n_2+r_2} \\
&\times \Bigg\{[J\cdot L_s(1+D_r) + J_1\cdot L_r(1-D_s)] \exp(i\lambda t\mathbf{a}_-(n_1,n_2)) \\
&\times \exp\left(-\frac{t}{2\gamma}\mathbf{b}_+(n_1,n_2)\right) + [J\cdot L_s(1-D_r) - J_1\cdot L_r(1-D_s)] \\
&\times \exp(-i\lambda t\mathbf{a}_+(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{c}_-(n_1,n_2)\right) \\
&+ [-J\cdot L_s(1+D_r) + J_1\cdot L_r(1+D_s)] \exp(i\lambda t\mathbf{a}_+(n_1,n_2)) \\
&\times \exp\left(-\frac{t}{2\gamma}\mathbf{c}_+(n_1,n_2)\right) + [-J\cdot L_s(1-D_r) - J_1\cdot L_r(1+D_s)] \\
&\times \exp(-i\lambda t\mathbf{a}_-(n_1,n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{b}_-(n_1,n_2)\right)\Bigg\} \\
&+ \cos\frac{\theta}{2}\sin\frac{\theta}{2} e^{i\phi} Q_{n_1+s_1} Q_{n_2+s_2} Q^*_{n_1+r_1+k_1} Q^*_{n_2+r_2+k_2}
\end{aligned}
$$

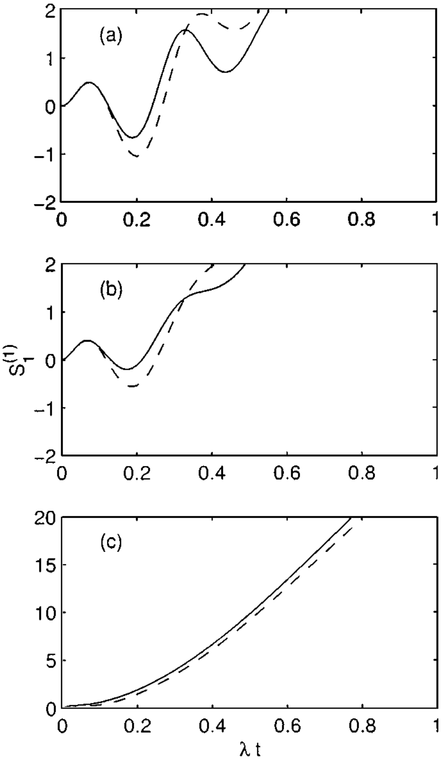

Fig. 7. Time evolution for the amplitude-squared squeezing parameter $S_1^{(1)}$ of Eq. (58) for $\theta = 0$ (the particle initially in the excited state) with the initial mean numbers $\bar{n}_1 = \bar{n}_2 = 10$, and the detuning parameter $\frac{\Delta}{2\lambda} = 0$ (broken curve) and 5 (full curve) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-2}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-1}$.

$$\times \Big\{ [J \cdot L_r(1 + D_s) + J_1 \cdot L_s(1 - D_r)] \exp(i\lambda t \mathbf{a}_-(n_1, n_2))$$

$$\times \exp\left(-\frac{t}{2y}\mathbf{b}_+(n_1, n_2)\right) + [-J \cdot L_r(1 + D_s) + J_1 \cdot L_s(1 + D_r)]$$

$$\times \exp(-i\lambda t \mathbf{a}_+(n_1, n_2)) \exp\left(-\frac{t}{2\gamma}\mathbf{c}_-(n_1, n_2)\right)$$

$$+ [J \cdot L_r(1 - D_s) - J_1 \cdot L_s(1 - D_r)] \exp(i\lambda t \mathbf{a}_+(n_1, n_2))$$

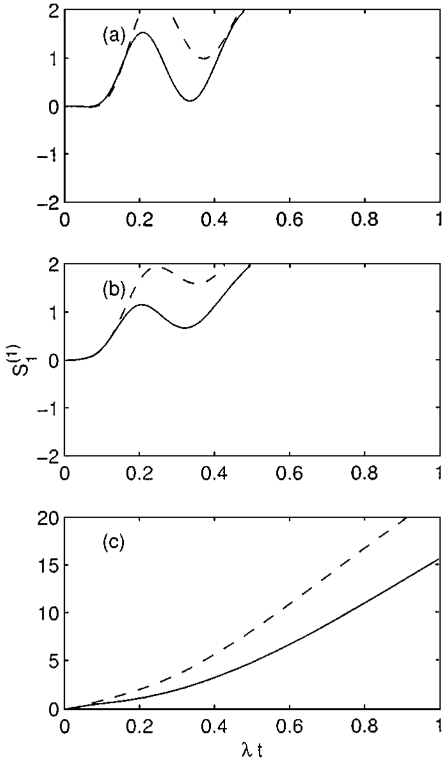

Fig. 8. Same as Fig. 7 but with $\theta = \pi$ (the particle initially in the ground state).

$$\times \exp\left(-\frac{t}{2\gamma}\mathbf{c}_{+}(n_1, n_2)\right) + [-J \cdot L_r(1 - D_s) - J_1 \cdot L_s(1 + D_r)]$$

$$\times \exp(-i\lambda\ t\mathbf{a}_{-}(n_1, n_2))\ \exp\left(-\frac{t}{2\gamma}\mathbf{b}_{-}(n_1, n_2)\right)\Bigg\}\Bigg] \tag{62}$$

where

$$J = \sqrt{\frac{(n_1 + s_1)!(n_2 + s_2)!}{(n_1)!(n_2)!}\frac{(n_1 + r_1)!(n_2 + r_2)!}{(n_2)!(n_2)!}};$$

$$J_1 \text{ is } J \text{ with } n_1 \to n_1 + k_1, n_2 \to n_2 + k_2 \tag{63}$$

$$L_s = \frac{\nu(n_1 + s_1 + k_1, n_2 + s_2 + k_2)}{\mu(n_1 + s_1 + k_1, n_2 + s_2 + k_2)}, \quad L_1 = L_s \times L_r$$

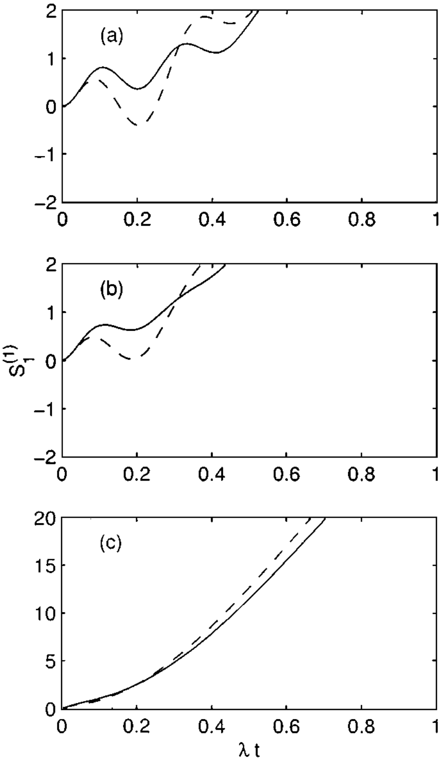

Fig. 9. Time evolution for the amplitude-squared squeezing parameter $S_1^{(1)}$ of Eq. (58) for $\theta = \frac{\pi}{4}$ and the relative phase $\phi = 0$ with the initial mean numbers $\bar{n}_1 = \bar{n}_2 = 10$ and the detuning parameter $\frac{\Delta}{2\lambda} = 0$ (broken curve) and 5 (full curve) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-2}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-1}$.

and

$$D_s = \frac{\left(\frac{\Delta}{2\lambda}\right)}{\mu(n_1 + s_1 + k_1, n_2 + s_2 + k_2)} \tag{64}$$

with

$$\begin{aligned} \mathbf{a}_{\pm}(n_1, n_2) = \mu(n_1 + r_1 + k_1, n_2 + r_2 + k_2) \\ \pm \mu(n_1 + s_1 + k_1, n_2 + s_2 + k_2) \end{aligned} \tag{65}$$

$$\mathbf{b}_{\pm}(n_1, n_2) = \{(\omega_1(r_1 - s_1) + \omega_2(r_2 - s_2) \pm \lambda \mathbf{a}_{-}(n_1, n_2))\}^2 \tag{66}$$

$$\mathbf{c}_{\pm}(n_1, n_2) = \{(\omega_1(r_1 - s_1) + \omega_2(r_2 - s_2) \pm \lambda \mathbf{a}_{+}(n_1, n_2))\}^2. \tag{67}$$

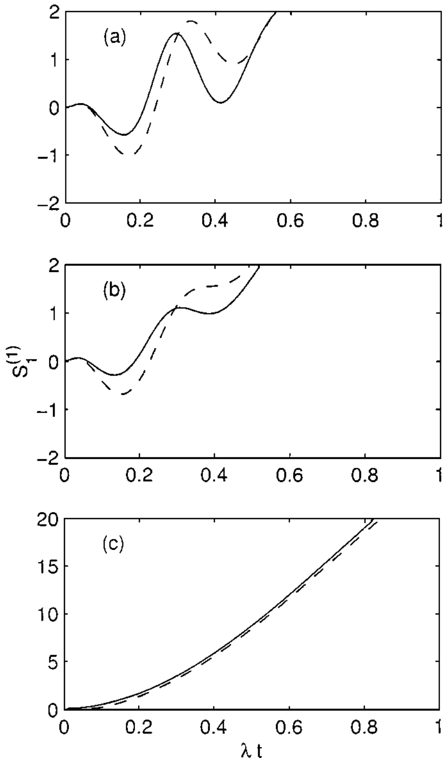

Fig. 10. Same as Fig. 9 but with the relative phase $\phi = \frac{\pi}{2}$.

Now we proceed to investigate the effect of the intrinsic decoherence on the amplitude-squared squeezing by using Eqs. (62–67), and specifying the exponents r, s we get the expression for the amplitude-squared squeezing of the first mode for two-quanta ($k_1 = k_2 = 1$).

Numerical results for Eq. (58) are presented in Figs. 7–12. Here we plotted amplitude-squared squeezing $S_1^{(1)}$ [Eq. (58)] against λt for $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the decoherence parameter $\frac{\lambda^2}{\gamma}$ (namely 10^{-6}, 10^{-2}, 10^{-1}) and different values of θ (namely 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, and π), and also the relative phase ϕ (namely 0 and $\frac{\pi}{2}$) either in the resonant or in the off-resonant cases.

In Figs. 7 and 8 we display amplitude-squared squeezing [Eq. (58)] with $\frac{\Delta}{2\lambda} = 0$ (broken curve) and $\frac{\Delta}{2\lambda} = 5$ (full curve), with different values of the decoherence parameter $\frac{\lambda^2}{\gamma}$ (namely 10^{-6}, 10^{-2}, 10^{-1}) for $\theta = 0$ (the particle initially in excited state) and $\theta = \pi$ (the particle initially in ground state), respectively. The first case $\theta = 0$ (the particle initially in the excited state) coincides with that of

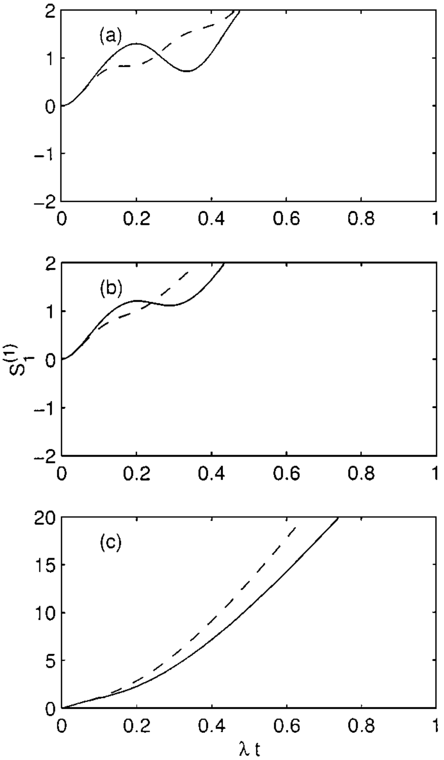

Fig. 11. Time evolution for the amplitude-squared squeezing parameter $S_1^{(1)}$ of Eq. (58) for $\theta = \frac{\pi}{2}$ and the relative phase $\phi = 0$ with the initial mean numbers $\bar{n}_1 = \bar{n}_2 = 10$ and the detuning parameter $\frac{\Delta}{2\lambda} = 0$ (broken curve) and 5 (full curve) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-2}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-1}$.

Obada *et al.* (1999). Also it is apparent from the calculations that for these special cases, phase does not affect squeezing.

In Figs. 9 and 10 we display the results with angle $\theta = \frac{\pi}{4}$ and for the values of relative phase (namely 0 and $\frac{\pi}{2}$), respectively with $\frac{\Delta}{2\lambda} = 0$ (broken curve) and $\frac{\Delta}{2\gamma} = 5$ (full curve) for different values of the decoherence parameter $\frac{\lambda^2}{\gamma}$ (namely 10^{-6}, 10^{-2}, 10^{-1}).

From Figs. 9 and 10 it is clear that in the absence of the phase $\phi = 0$ the squeezing does not appear in the off-resonant case $\frac{\Delta}{2\lambda} = 5$ (full curve) (see Fig. 9), and in the resonant case $\frac{\Delta}{2\lambda} = 0$ (broken curve) it appears just when $\frac{\lambda^2}{\gamma} = 10^{-6}$ (see Fig. 9 (a)), while in presence of the phase the squeezing appears in two cases $\frac{\Delta}{2\lambda} = 0$ and $\frac{\Delta}{2\lambda} = 5$ (see Fig. 10).

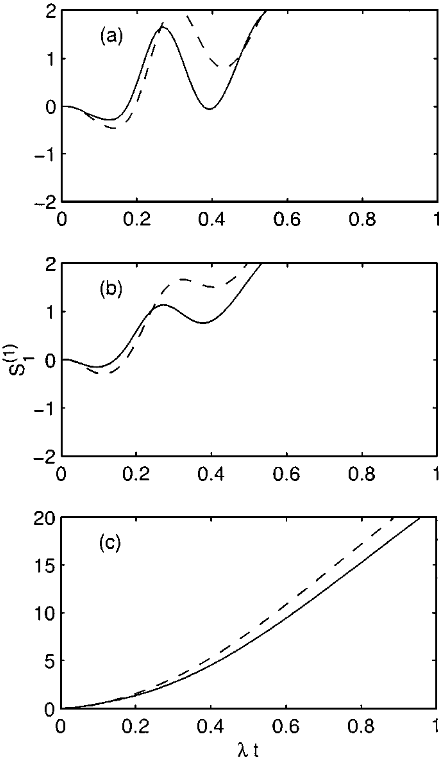

Fig. 12. Same as Fig. 11 but with the relative phase $\phi = \frac{\pi}{2}$.

Figures 11 and 12 are same as figures 9 and 10, but $\theta = \frac{\pi}{2}$. We see from these figures no squeezing occurs in two cases—resonant and off-resonant—for $\phi = 0$ (see Fig. 11), and when $\phi = \frac{\pi}{2}$ it occurs (see Fig. 12), but the squeezing in these cases (Fig. 12) is less than those in Fig. 10.

In view of these Figs. 7–12, we observe that the amount of squeezing rapidly decrease with the decrease of the decoherence parameter γ. Thus we conclude that the effect of the intrinsic decoherence on the squeezing of the field is to suppress the squeezing of the field in the nondegenerate multiquanta JCM.

4. CONCLUDING REMARKS

In this paper, we have considered an effective Hamiltonian (6) describing the interaction between a two-level particle (atom or trapped ion) and a two-mode field through multiquanta. The appearance of this Hamiltonian in trapped ion experiments in the Lamb-Dicke regime and with suitable side-band detunings has been mentioned. We have found the exact solution of the Milburn equation [Eq. (5)] for the nondegenerate multiquanta JCM. Using the exact solution [Eq. (51)], we have discussed the effect of the intrinsic decoherence on population inversion and squeezing of the radiation field. It is shown that the intrinsic decoherence in the particle-field interaction suppress the nonclassical effects, where with the decrease of the parameter γ, i.e. with a more rapid decohering, we observed a rapid decrease of the amount of squeezing. The detuned model is more susceptible to decoherence. It is also to be remarked that the phenomena depending on the number operators (such as mean values of number of quanta and population inversion) show more rapid decoherence than those depending upon A, $A^{\dagger}$ or their powers.

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